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CALCULATION OF FACTORIALS

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CALCULATION OF FACTORIALS

SUMMARY:

Description of a method for calculating terms containing products and reciprocals of factorials.

INTRODUCTION:

In calculating functions employed in celestial mechanics, it is often necessary to evaluate terms such as $x!/m!(x-m)!$. Direct computer calculation of such a term is time consuming and results either in overflow or loss of significance. However, by using the canonical decomposition of each factor of the factorial, rather than the factorial itself, computation of such terms can be reduced to a few steps involving only addition and subtraction of exponents.

DEVELOPMENT:

In the implementation of this method, each factorial is represented by a positional notation of the exponents of its prime factors. $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$ is represented as $2^3 \cdot 3^1 \cdot 5^1$. A base logic word, $L_b = 50505050$, is then chosen and the exponents of the primes are introduced into it. Factorial 5 is now represented as 53515150. The exponent of two is carried in the first two positions, the exponent of 3 in the 3rd and 4th position, etc.

Calculation with these logic words consists of logical addition of one word to another, $L_1 + L_2 - L_b$ and logical subtraction of one word from another, $L_1 + L_B - L_2$. These two operations represent, respectively, multiplication and division of factorials.

As an example, the number representing $10!/4!8!$ could be calculated by logically subtracting the logic word representing 4! from the logic word representing 10! and then subtracting the logic word representing 8! from the result of the prior operation.

Representation:

Factorial		Logic Word		Powers of Primes
10!	=	58545251	=	$2^8 \cdot 3^4 \cdot 5^2 \cdot 7^1$
8!	=	57525151	=	$2^7 \cdot 3^2 \cdot 5^1 \cdot 7^1$
4!	=	53515050	=	$2^3 \cdot 3^1 \cdot 5^0 \cdot 7^0$

Multiplication: (4!)(8!)

L_1	53515050
+ L_2	+ 57525151
=	111040201
- L_B	- 50505050
=	60535151

Division: 10!/(4!8!)

L_1	58545251
+ L_B	50505050
=	109050301
- L_2	60535151
=	48515150

$$48515150 = 2^{-2} \cdot 3^1 \cdot 5^1 \cdot 7^0 = \frac{15}{4}$$

The result of these operations can be used in the logical form or the positive and negative exponents can be evaluated separately and the result presented as a rational fraction in which numerator and denominator are relatively prime.

In the program using this technique, three logic words were employed to represent each factorial. This allowed positions for exponents of the first twelve primes and factorials to 37! could be represented. Larger factorials can be represented by base logic word enlargement (e.g., to 500500500) and increasing the number of logic words.

CONCLUSION:

This method allows one to accurately calculate with factorials wherein intermediate values generated by direct calculation might cause overflow or loss of significance. In addition, the technique is easy to use and is more rapid than direct calculation.